

Trends for Reducing Global Warming

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We know that Global Warming can be reduced during the years of the century ahead of us if we — our civilization — steadily reduces its emissions of carbon dioxide gas (CO₂) into the atmosphere.

Given a specific rate for the reduction of anthropogenic (our CO₂) emissions:

— how long will it take to return Earth's average temperature to its unperturbed pre-industrial level?, and

— how much higher will Global Warming (Earth's temperature) become before it begins to decrease?

Answering these questions is the subject of this study. My work here is based on a Carbon Balance Model, which I described in an earlier report:

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A Carbon Balance Model of Atmospheric CO₂

11 September 2020

<https://manuelgarciajr.files.wordpress.com/2020/09/a-carbon-balance-model-of-atmospheric-co2.pdf>

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That model has been further refined in order to address these new questions, and the details of that refinement are described in the present report.

Prior to the buildup of anthropogenic CO₂ emissions in the air, the fluxes of CO₂ released by the respiration of Life-on-Earth; and the fluxes of CO₂ absorbed from the air by photosynthesis, the

surface waters of the oceans, and rock weathering chemical reactions; were in balance. That balance is known as the Carbon Cycle.

As the rate and buildup of anthropogenic emissions increased (after ~1750, but particularly from the mid-20th century), the Carbon Cycle was perturbed out of balance, and the magnitude of that imbalance is determined by the difference between two effects: Anthropogenic Sources, and Stimulated Sinks.

The Anthropogenic Sources are:

- the CO₂ emissions by the human activities of fossil-fueled energy generation and industry, and
- the CO₂ emissions from land use changes (deforestation and its attendant increase of wildfires).

The Stimulated Sinks are the *additional* absorption of CO₂ by photosynthesis and the surface waters of the oceans, because of higher atmospheric concentrations of CO₂. At a sufficiently high level of atmospheric CO₂ concentration, both these sinks will saturate — stop absorbing CO₂. What that “sufficiently high level” is remains uncertain.

The present study includes more realistic (more complicated) models of these source and sink terms in the rate equation for the change of the Carbon Balance over time.

The rest of this report shows the physics and math details of the above.

SOURCE

Anthropogenic CO₂ emissions in year 2020 are 42.2GtCO₂/y (42.2 giga-metric-tons of CO₂ per year = $42.2 \cdot 10^{+12}$ kilograms/year). This magnitude of total anthropogenic emissions, E, is the

addition of our fossil-fueled and land use emissions.

Assuming that humanity woke up and immediately began reducing E exponentially after year y_1 , the time dependence (in years) of $E(y)$ would be:

$$E(y) = E_1 * e^{-(y-y_1)/\Delta t_3},$$

for:

$$E_1 = E(y_1),$$

Δt_3 = characteristic decay time of $E(y)$.

[The subscript "3" is used here to avoid confusion with the Δt_1 and Δt_2 in the previous report. That previous Δt_2 will be used in this report as well.]

In my previous report, I only considered linearly increasing and decreasing trends of $E(y)$.

How specific trends of the exponential type affect the course of Global Warming will be shown later.

UNITS

I use the following units conversions (see the previous report for how they were arrived at):

$$1\text{ppm}(\text{CO}_2) = 8.16903\text{GtCO}_2,$$

[ppm = parts-per-million (by volume) of atmospheric concentration],

$$\pm 130\text{ppm}(\text{CO}_2) = \pm 1^\circ\text{C},$$

[average global temperature increase (if +) or decrease (if -)].

For example,

$$E(2020) = +42.2\text{GtCO}_2/\text{y} = \\ = +5.17\text{ppm}(\text{CO}_2)/\text{y} = +0.0397^\circ\text{C},$$

as the source flux during year 2020.

The sink will absorb some of this emission, and the net contribution from year 2020 to the CO₂ accumulation in the atmosphere will be less than E(2020), (being $\sim 1/2$).

Another conversion factor is:

$$1\text{GtC} = 3.664\text{GtCO}_2,$$

where GtC is the unit of giga-metric-tons of carbon. A carbon dioxide molecule is 3.664x more massive ("heavier") than a carbon atom.

SINK

Sibbele Hietkamp (of South Africa) has sent me data on the sink (the perturbation absorption rate) for the years 1959 through 2020. The temporal variation of that sink can be characterized by a linear relation (an assumption), as:

$$S_h(y) = \frac{(y-1959)}{15.25} + 0.8 \pm 2\text{GtC},$$

in units of GtC.

TABLE 1: Hietkamp Sink, and NOAA ppm Data

YEAR	CO2 ppm, NOAA (1960-2020)	$S_h \pm 2,$ GtC/y	$S_h \pm 0.897,$ CO2ppm/y
1946.8	307, (MG,Jr.)		0.000, (MG,Jr.)
1959	316, (MG,Jr.)	0.8	0.359
1960	317	0.866	0.388
1970	325	1.521	0.683
1980	340	2.177	0.977
1990	352	2.833	1.271
2000	368	3.489	1.565
2010	387	4.144	1.859
2020	412	4.8	2.153

I have cross-correlated $S_h(y)$ and $C(y)$ (the atmospheric CO2 concentration at year y) for the years 1959 to 2020, to capture the effect of $C(y)$ stimulating (enhancing) S_h as $C(y)$ increases: $S_h(C)$,

$$S_h(C) = 2.2295 * \ln(C - 307 \text{ ppm}) - 5.6296, \text{ GtC},$$

for:

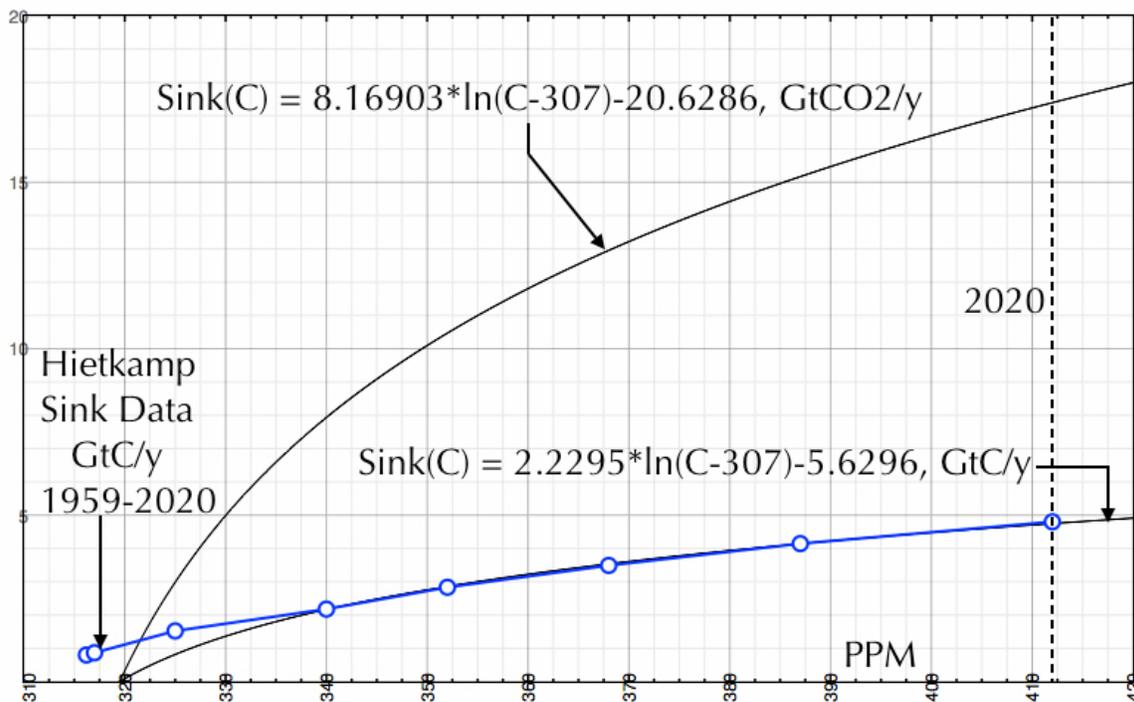
C = the atmospheric CO2 concentration in ppm.

In different units:

$$S_h(C) = 8.169 * \ln(C - 307 \text{ ppm}) - 20.628, \text{ GtCO}_2/\text{y},$$

$$S_h(C) = 1 * \ln(C - 307 \text{ ppm}) - 2.525, \text{ ppm(CO}_2)/\text{y}.$$

Notice that the sink varies as the logarithm of C .



This logarithmic sink function applies for $C \geq 319.491$ ppm. Obviously, it follows the sink data very closely for $C > 340$ ppm.

The trend toward saturation of S_h at high C is implicit in the logarithmic relationship between them.

In my previous report, I only considered the sink as linearly proportional to C without limit (no saturation).

RATE EQUATION FOR CARBON BALANCE

Given the above for the source and sink, and continuing with the decay rate by weathering used in the previous report, the rate equation for the carbon balance is now:

$$\frac{dC(y)}{dy} = E_1 * e^{-(y-y_1)/\Delta t_3} - [1 * \ln(C-307\text{ppm}) - 2.525] +$$

$$- (C_1 / \Delta t_2) * e^{-(y-y_1)/\Delta t_2},$$

for

$$E_1 = E(y_1),$$

$$C_1 = C(y_1),$$

$$\Delta t_2 = 1442.7y,$$

and where units of ppm/y are used.

Defining:

$$A = [C-307], \text{ ppm},$$

$$A_0 = 12.491\text{ppm},$$

$$x = [y-y_1], \text{ years with respect to } y_1,$$

the rate equation appears as

$$\frac{dA(x)}{dx} = E_1 * e^{-x/\Delta t_3} - 1 * \ln[A/A_0] +$$

$$- (C_1 / \Delta t_2) * e^{-x/\Delta t_2},$$

[the "1" carries units of ppm/y, the argument of the \log_e is dimensionless].

Defining:

$z = x/x_0$, a dimensionless time variable scaled by time (years)
 x_0 ,

$$D = (C_1/\Delta t_2), \text{ ppm/y}$$

(the initial magnitude of the long-term decay function), then

$$(A_0/x_0)^{d[A/A_0]}/dz = E_1 * e^{-(x_0 * z)/\Delta t_3} - 1 * \ln[A/A_0] + \\ - D * e^{-(x_0 * z)/\Delta t_2},$$

and defining:

$$B = A/A_0, \text{ (dimensionless dependent variable),}$$

$$R_0 = (A_0/x_0), \text{ ppm/y, (a constant),}$$

$$\beta_3 = x_0/\Delta t_3, \text{ (a dimensionless constant),}$$

$$\beta_2 = x_0/\Delta t_2, \text{ (a dimensionless constant),}$$

$$\mu_1 = 1/(A_0/x_0), \text{ (a dimensionless constant),}$$

$$\mu_3 = E_1/(A_0/x_0), \text{ (a dimensionless constant),}$$

$\mu_2 = D/(A_0/x_0)$, (a dimensionless constant),

the rate equation appears in the following form:

$$d[B]/dz = \mu_3 * e^{-(\beta_3 * z)} - \mu_1 * \ln[B] - \mu_2 * e^{-(\beta_2 * z)},$$

or

$$d[B]/dz + \mu_1 * \ln[B] = \mu_3 * e^{-(\beta_3 * z)} - \mu_2 * e^{-(\beta_2 * z)},$$

which is a nondimensional first order nonlinear differential equation for B(z).

I have not been able to solve this equation in closed form (exactly in an analytical fashion), so what follows is an iterative solution.

ITERATIVE SOLUTION

B_0 = a judiciously chosen 0th iterant (a constant), which characterizes the magnitude of B(z) over the range of interest. The rate equation is written for 1st iterant, $B_1(z)$, as

$$d[B_1(z)]/dz = -\mu_1 * \ln[B_0] + \mu_3 * e^{-(\beta_3 * z)} - \mu_2 * e^{-(\beta_2 * z)},$$

and integrated to produce

$$B_1(z) = B(0) - \mu_1 * \ln[B_0] * z + (\mu_3 / \beta_3) * [1 - e^{-(\beta_3 * z)}] + \\ - (\mu_2 / \beta_2) * [1 - e^{-(\beta_2 * z)}],$$

where

$$B(0) = [C(y_1)^{-307}] / A_0.$$

The maximum or minimum value of $B_1(z)$ is found, as well as the z coordinate at that location, z_1 .

$$0 = -\mu_1 * \ln[B_0] + \mu_3 * e^{-(\beta_3 * z_1)} - \mu_2 * e^{-(\beta_2 * z_1)}.$$

For "short" time spans of only a century or two

$$\beta_2 * z_1 \ll 1, \text{ and}$$

$$0 = -\mu_1 * \ln[B_0] + \mu_3 * e^{-(\beta_3 * z_1)} - \mu_2 * [1 - \beta_2 * z_1 + \dots],$$

$$0 \approx -\mu_1 * \ln[B_0] + \mu_3 * e^{-(\beta_3 * z_1)} - \mu_2,$$

and from this we find

$$z_1 = (-1 / \beta_3) * \ln[(\mu_1 * \ln(B_0) + \mu_2) / \mu_3],$$

and the maximum or minimum value, $B_1(z_1)$, is

$$B_1(z_1) = B(0) - \mu_1 * \ln[B_0] * z_1 + (\mu_3/\beta_3) * [1 - e^{-(\beta_3 * z_1)}] +$$

$$- (\mu_2/\beta_2) * [1 - e^{-(\beta_2 * z_1)}].$$

For the constant used (in the logarithm) to characterize the magnitude of function $B_1(z)$ in the equation for the 2nd iterant, $B_2(z)$, I choose

$$B_1 = B(0) + (1/2) * [B_1(z_1) - B(0)].$$

The 2nd iterant is

$$B_2(z) = B(0) - \mu_1 * \ln[B_1] * z + (\mu_3/\beta_3) * [1 - e^{-(\beta_3 * z)}] +$$

$$- (\mu_2/\beta_2) * [1 - e^{-(\beta_2 * z)}].$$

One can continue this sequence of iterations to arrive at whatever degree of precision is desired. Given the limitations on the accuracy of the data, and of the model itself, I will stop iterating here, and calculate specific cases. First, however, I will give a summary of the solution.

SUMMARY OF THEORETICAL RESULTS

$$B_2(z) = B(0) - \mu_1 * \ln[B_1] * z + (\mu_3/\beta_3) * [1 - e^{-(\beta_3 * z)}] +$$

$$- (\mu_2/\beta_2) * [1 - e^{-(\beta_2 * z)}].$$

$$B_2(z) = [C(z)-307]/A_0,$$

$$B(0) = [C(y_1)-307]/A_0,$$

$$A_0 = 12.491\text{ppm},$$

$$R_0 = (A_0/x_0), \text{ ppm/y, (a constant),}$$

$$\beta_3 = x_0/\Delta t_3, \text{ (a dimensionless constant),}$$

$$\beta_2 = x_0/\Delta t_2, \text{ (a dimensionless constant),}$$

$$\mu_1 = 1/(A_0/x_0), \text{ (a dimensionless constant),}$$

$$\mu_3 = E_1/(A_0/x_0), \text{ (a dimensionless constant),}$$

$$\mu_2 = D/(A_0/x_0), \text{ (a dimensionless constant),}$$

$$\Delta t_2 = 1442.7\text{y},$$

$$E_1 = E(y_1) \text{ ppm/y},$$

$$D = (C_1/\Delta t_2) \text{ ppm/y},$$

$$B_1(z_1) = B(0) - \mu_1 * \ln[B_0] * z_1 + (\mu_3/\beta_3) * [1 - e^{-(\beta_3 * z_1)}] +$$

$$- (\mu_2/\beta_2) * [1 - e^{-(\beta_2 * z_1)}],$$

$$z_1 = (-1/\beta_3) * \ln[(\mu_1 * \ln(B_0) + \mu_2)/\mu_3],$$

$$B_1 = B(0) + (1/2) * [B_1(z_1) - B(0)],$$

B_0 = 0th iterant, a constant (a good guess is needed),

x_0 = a time span for scaling (normalizing/non-dimensionalizing)
the independent variable $x=(y-y_1)$,

$$z = x/x_0,$$

$$z = (y-y_1)/x_0,$$

Δt_3 = the characteristic decay time for the source term $E(x)$
(anthropogenic emissions).

CASE #1, $\Delta t_3=40y$

$$y_1 = 2020$$

$$x_0 = 10y$$

$$R_0 = 1.2491\text{ppm/y}$$

$$E_1 = 5.166 \text{ ppm/y, (42.2 GtCO}_2\text{/y)}$$

$$C_1 = 417 \text{ ppm}$$

$$D = 0.289 \text{ ppm/y}$$

$$\mu_1 = 0.80058$$

$$\mu_3 = 4.1361$$

$$\mu_2 = 0.231$$

$$\beta_3 = 0.25$$

$$\beta_2 = 0.00693$$

$$B(0) = 8.80634, (C(y_1)=417 \text{ ppm})$$

$$B_0 = 10, (C=431.9).$$

Using the above, the following are found:

$$z_1 = 2.76023,$$

(this z_1 corresponds to $y=2047.6$; in 27.6 years),

$$B_1(z_1) = 11.336,$$

$$B_1 = 10.07137,$$

$$\begin{aligned}
B_2(z) &= 8.80634 - (0.8) \cdot \ln[10.07137] \cdot z + \\
&+ (16.544) \cdot [1 - e^{-(0.25 \cdot z)}] + \\
&- (33.33) \cdot [1 - e^{-(0.00693 \cdot z)}] = \\
&= [C(z) - 307] / (12.491 \text{ ppm})
\end{aligned}$$

$$\begin{aligned}
C(x) &= 417 \text{ ppm} - (2.31 \text{ ppm/y}) \cdot x + \\
&+ (206.65 \text{ ppm}) \cdot (1 - e^{-0.025 \cdot x}) + \\
&- (416.3 \text{ ppm}) \cdot (1 - e^{-0.000693 \cdot x}).
\end{aligned}$$

The temperature excursion associated with this concentration history is

$$\begin{aligned}
\Delta T(x) &= 3.208 - 0.01777 \cdot x + 1.5896 \cdot (1 - e^{-0.025 \cdot x}) + \\
&- 3.20254 \cdot (1 - e^{-0.000693 \cdot x}) - 2.131, \text{ }^\circ\text{C}.
\end{aligned}$$

CASE #2, $\Delta t_3 = 100\text{y}$

$$\beta_3 = 0.1,$$

$$\Delta t_3 = 100\text{y},$$

and all the other parameters are the same as in CASE #1. Thus:

$$z_1 = 6.9$$

(which corresponds to $y=2089$; in 69y),

$$B_1(z_1) = 15.155,$$

$$B_1 = 11.981,$$

and:

$$\begin{aligned} C(x) = & 417\text{ppm} - (2.482\text{ppm/y}) * x + \\ & + (516.64\text{ppm}) * (1 - e^{-0.01 * x}) + \\ & - (416.3\text{ppm}) * (1 - e^{-0.000693 * x}). \end{aligned}$$

The temperature excursion associated with this concentration history is

$$\begin{aligned} \Delta T(x) = & 3.208 - 0.01909 * x + 3.9742 * (1 - e^{-0.01 * x}) + \\ & - 3.20254 * (1 - e^{-0.000693 * x}) - 2.131, \text{ } ^\circ\text{C}. \end{aligned}$$

CASE #3, $\Delta t_3=200y$

$$\beta_3 = 0.05$$

$$z_1 = 13.814,$$

(this coordinate corresponds to $y=2158$; in 138y),

$$B_1(z_1) = 21.577,$$

$$B_1 = 15.192,$$

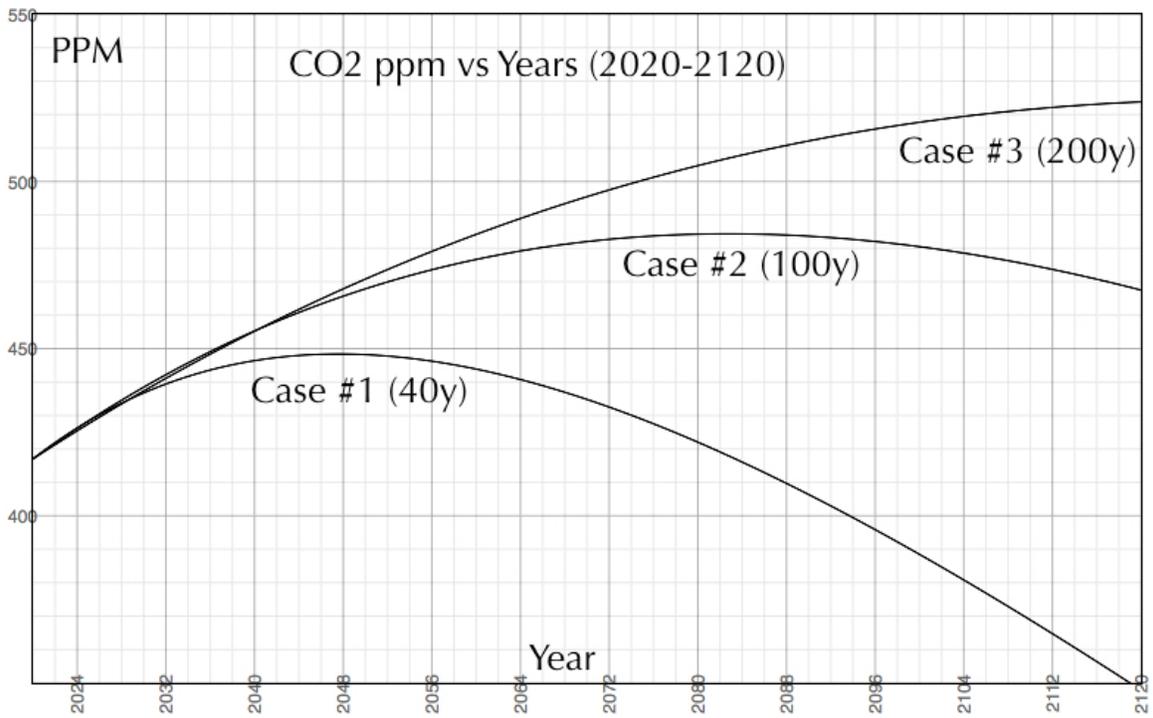
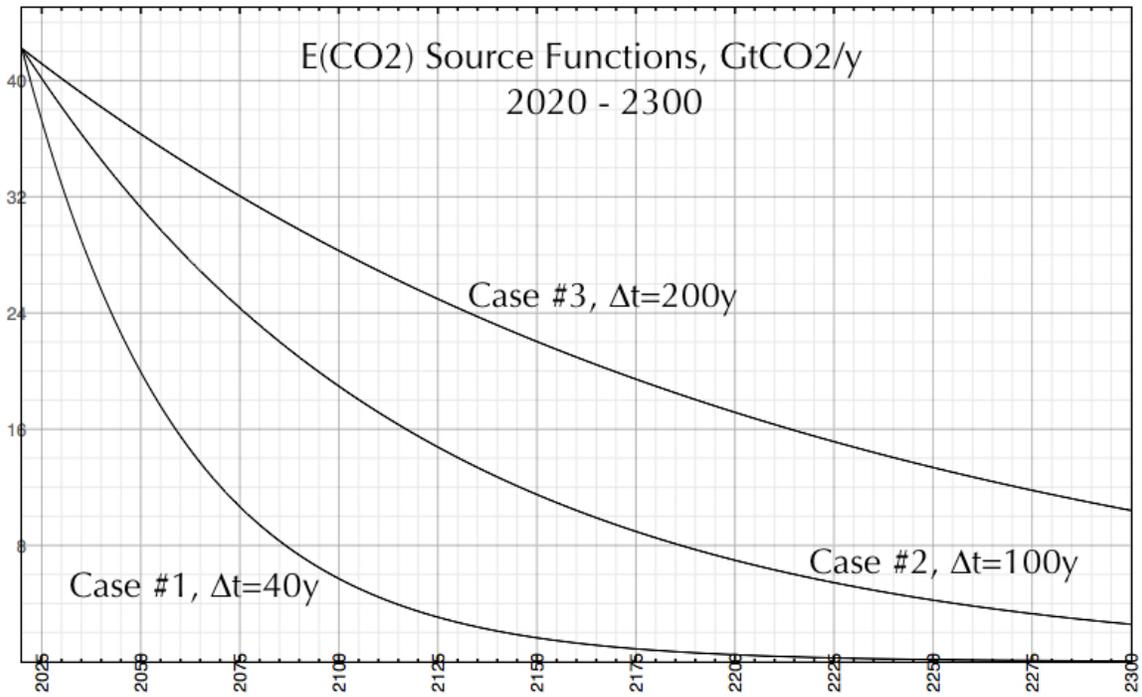
and:

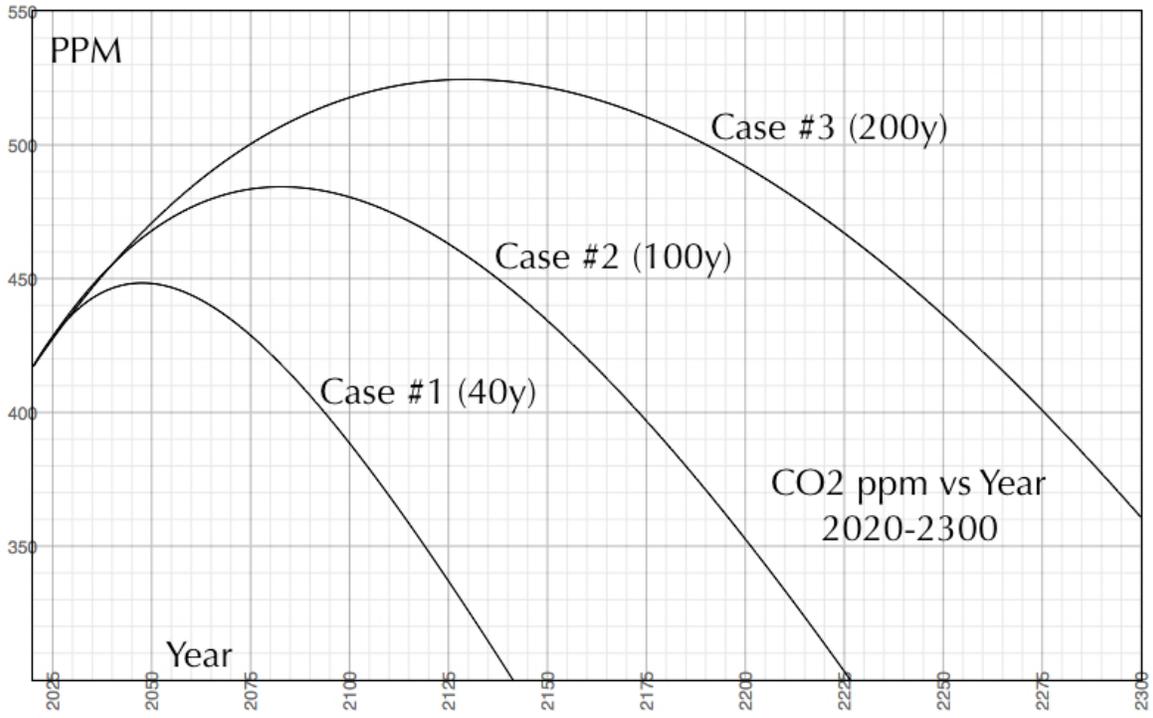
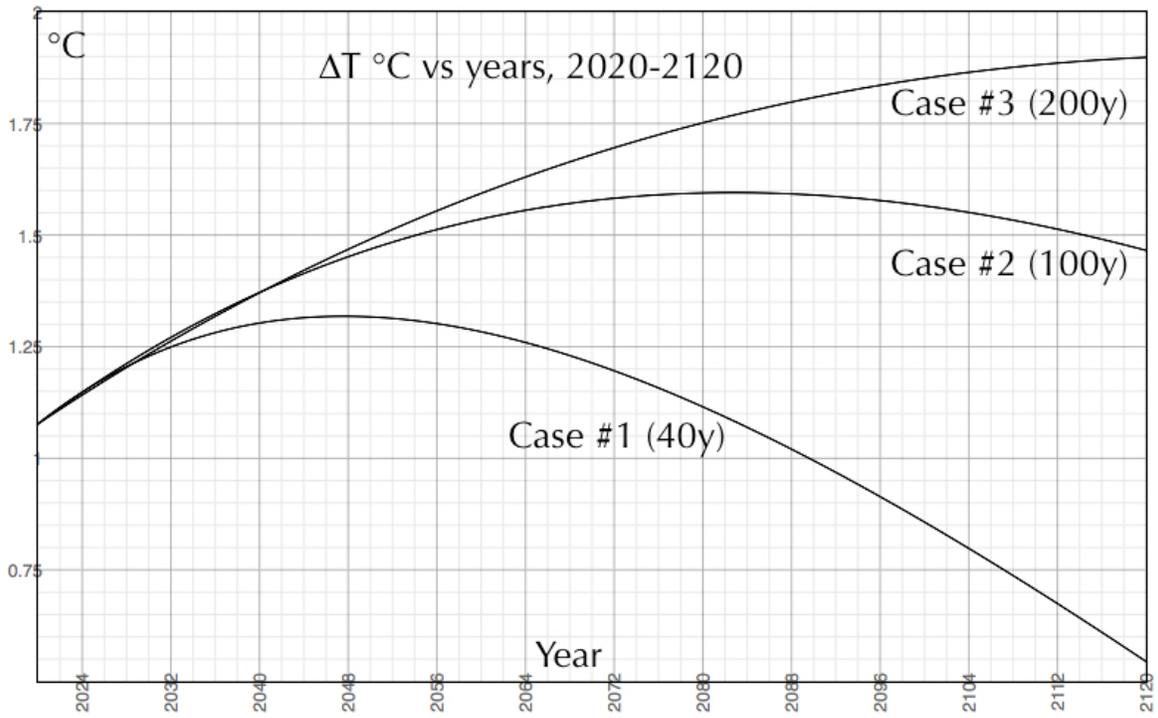
$$C(x) = 417\text{ppm} - (2.719\text{ppm/y}) * x + \\ + (1033.3\text{ppm}) * (1 - e^{-0.005 * x}) + \\ - (416.3\text{ppm}) * (1 - e^{-0.000693 * x}).$$

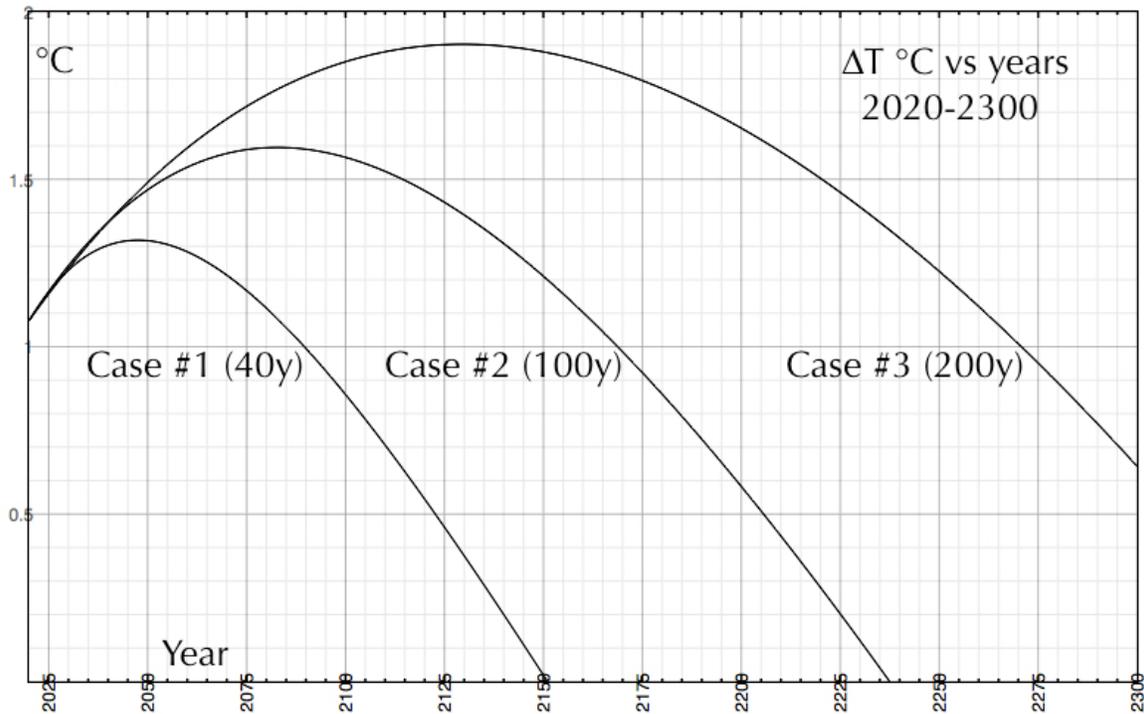
The temperature excursion associated with this concentration history is

$$\Delta T(x) = 3.208 - 0.02092 * x + 7.9485 * (1 - e^{-0.005 * x}) + \\ - 3.20254 * (1 - e^{-0.000693 * x}) - 2.131, \text{ } ^\circ\text{C}.$$

GRAPHS







CASE #1, $\Delta t_3 = 40y$:

This trend reaches a peak of 449ppm and +1.32°C in year 2048 (in 28 years); it remains above 440ppm and +1.25°C over the years 2032 to 2064 (between 12 and 44 years from now); then descends to 350ppm and +0.56°C in year 2120 (in 100 years); and 300ppm and +0.18°C in year 2140 (in 120 years).

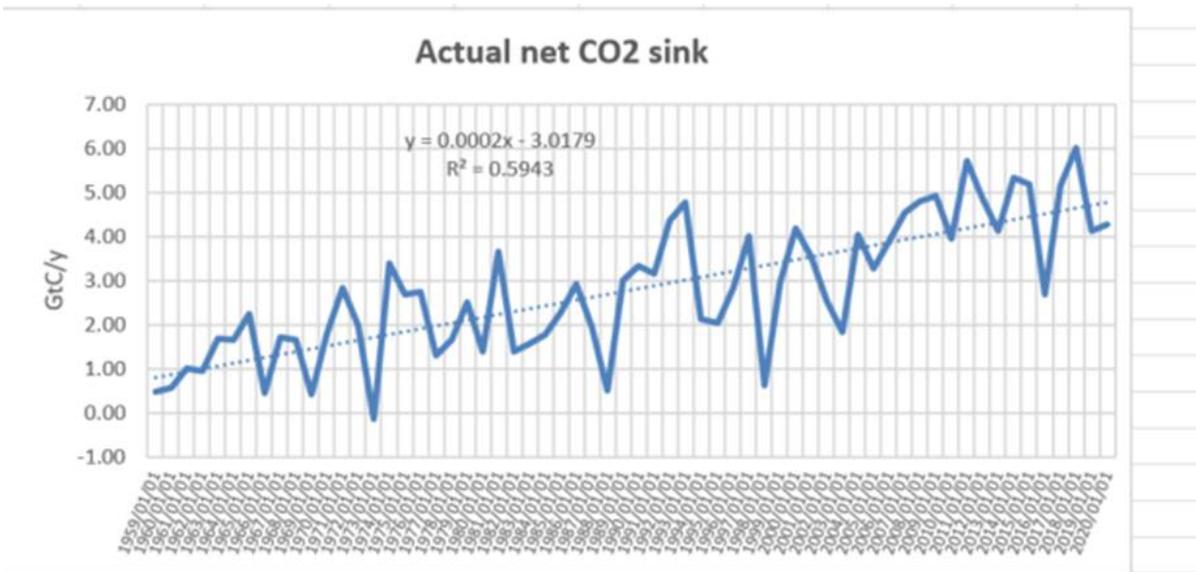
CASE #2, $\Delta t_3 = 100y$:

This trend reaches a peak plateau of 485ppm and +1.6°C over the years 2078 to 2088 (between 58 and 68 years from now); it remains above 480ppm and +1.56°C during years 2066 to 2100 (between 46 and 80 years from now); it descends to 350ppm and +0.56°C in year 2202 (in 182 years); and 300ppm and +0.18°C in year 2225 (in 205 years).

CASE #3, $\Delta t_3 = 200y$:

This trend reaches a peak plateau of 524ppm and +1.9°C over the years 2125 to 2135 (between 105 and 115 years from now); it remains above 500ppm and +1.72°C between years 2075 and 2190 (between 55 and 170 years from now); and descends down to 360ppm and +0.64°C in year 2300 (in 280 years).

APPENDIX: HIETKAMP SINK DATA



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